

Violation of Bell's inequality in electronic Mach-Zehnder interferometers

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Abstract

We propose a possible setup of testing the Bell's inequality in mesoscopic conductors. The particular implementation uses two coupled electronic Mach-Zehnder interferometers in which electrons are injected into the conductors in the quantum Hall regime. It is shown that the Bell's inequality is violated for an arbitrary coupling strength between the two interferometers.

Key words: , Bell's inequality, entanglement, nonlocality, electronic Mach-Zehnder interferometer

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One of the most striking features of quantum theory is the entanglement that gives rise to strong nonlocal correlations between spatially separated particles. The peculiar properties of entanglement are inconsistent with any local realistic theories and can be tested through Bell's inequality (BI) [1]. Experiments with entangled photon pairs have shown violation of the BI [2]. However, no violation of BI with electrons has been realized yet. Recent advance in nanofabrication technology makes it possible to study fundamental problems of quantum theory such as entanglement and measurement in mesoscopic devices.

In this paper, we propose a possible realization of the violation of BI in mesoscopic conductors. The schematic diagram of Fig. 1 represents two coupled electronic Mach-Zehnder interferometers [3] in which electrons are injected into the conductors in the quantum Hall regime. This setup has been considered for studying complementarity principle that enables a choice of wavelike or particlelike behavior of electrons [4]. A recent experiment has reported which-path (WP) detection and recovery of interference in correlation measurement in an electronic Mach-Zehnder interferometer by using the quantum

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Hall edge channels [5], which demonstrates that our scheme can also be implemented with current technology.

The two interferometers are electrostatically coupled to each other so that the Coulomb interaction in the contact region gives rise to modification of the trajectory leading to a phase shift (denoted by $\Delta\phi$). No electron exchange is allowed between the two interferometers. The phase shift $\Delta\phi$ makes an entangled state of the two electrons (one injected from the upper and the other from the lower interferometers) [6].

Let us consider a two electron injection process, one from lead $\bar{\alpha}$ and the other from $\bar{\gamma}$. Two types of electron creation operators are introduced, namely c_x^\dagger and b_x^\dagger . The operators c_x^\dagger and b_x^\dagger create an electron at lead x and at the intermediate regions, respectively. The beam splitter BS- i , made of a quantum point contact, is characterized by the scattering matrix S_i ($i = 1, 2, 3, 4$)

$$S_i = \begin{pmatrix} r_i & t'_i \\ t_i & r'_i \end{pmatrix}, \quad (1a)$$

which transforms the electron operators as

$$(c_\alpha^\dagger \ c_\beta^\dagger) = (b_\alpha^\dagger \ b_\beta^\dagger)S_1, \quad (b_\alpha^\dagger \ b_\beta^\dagger) = (c_\alpha^\dagger \ c_\beta^\dagger)S_2, \quad (1b)$$

$$(c_\gamma^\dagger \ c_\delta^\dagger) = (b_\gamma^\dagger \ b_\delta^\dagger)S_3, \quad (b_\gamma^\dagger \ b_\delta^\dagger) = (c_\gamma^\dagger \ c_\delta^\dagger)S_4. \quad (1c)$$

The injected two electron state prior to interaction at the intermediate region is written as

$$|\Psi_0\rangle = c_\alpha^\dagger c_\gamma^\dagger |0\rangle = (r_1 b_\alpha^\dagger + t_1 b_\beta^\dagger) \otimes (r_3 b_\gamma^\dagger + t_3 b_\delta^\dagger) |0\rangle, \quad (2)$$

where $|0\rangle$ is the ground state without injection. It is assumed that Coulomb interaction affects only the trajectory of the two electrons and inelastic scattering is neglected. This assumption can be justified at low bias and temperature. The modification of trajectory changes the area enclosed by the loops of the interferometers and induces an additional phase shift $\Delta\phi$

$$\Delta\phi = 2\pi H \Delta A / \Phi_0, \quad (3)$$

for the particular path in which both electrons are transmitted. H and ΔA stand for the external magnetic field and the area enclosed by the change of the trajectory resulting from the interaction (represented by the shaded region of Fig. 1), respectively. $\Phi_0 = hc/e$ is the flux quantum. As a result, the two-electron state upon a scattering can be written as

$$|\Psi\rangle = (r_1 b_\alpha^\dagger \chi_r^\dagger + t_1 b_\beta^\dagger \chi_t^\dagger) |0\rangle, \quad (4a)$$

where the operators χ_r^\dagger and χ_t^\dagger create the states of the lower interferometer depending on whether the electron in the upper interferometer is reflected or transmitted, respectively:

$$\chi_r^\dagger = r_3 b_\gamma^\dagger + t_3 b_\delta^\dagger, \quad (4b)$$

$$\chi_t^\dagger = r_3 b_\gamma^\dagger + t_3 e^{i\Delta\phi} b_\delta^\dagger. \quad (4c)$$

Eq. (4) describes an entanglement of the two interferometers. Because of the phase factor $e^{i\Delta\phi}$, $\chi_r^\dagger \neq \chi_t^\dagger$ and the extent of the entanglement can be controlled by changing H or ΔA .

The interference of single electrons in the upper interferometer is displayed in the probability of finding an electron at lead a ($\in \alpha, \beta$),

$$P_a = \langle \Psi | c_a^\dagger c_a | \Psi \rangle. \quad (5)$$

The evaluation with the help of Eqs. (1,4) gives

$$\begin{aligned} P_\alpha &= 1 - P_\beta \\ &= R_1 R_2 + T_1 T_2 + 2|\nu| \sqrt{R_1 T_1 R_2 T_2} \cos(\varphi - \phi_\nu), \end{aligned} \quad (6)$$

where $T_i = |t_i|^2$ and $R_i = |r_i|^2$ are the transmission and the reflection probabilities at BS- i , respectively. The overlap of the states $\nu \equiv \langle 0 | \chi_t \chi_r^\dagger | 0 \rangle$ is a measure of the WP information and $\phi_\nu \equiv \arg \nu$. The phase ϕ_1 enclosed by the loop of the upper interferometer is given as $\phi_1 = \arg(t_1) + \arg(t'_2) - \arg(r_1) - \arg(r_2)$.

Eq. (6) shows the relation between the interference fringe of the upper and the WP information stored in the lower interferometer. If the two states $\chi_r^\dagger | 0 \rangle$ and $\chi_t^\dagger | 0 \rangle$ are orthogonal (that is $\nu = 0$), then a complete WP information is acquired and the interference disappears in the upper interferometer. Complete WP information can be obtained for a symmetric BS-3 ($|r_3| = |t_3| = 1/\sqrt{2}$) with $\Delta\phi = \pi$. Note that ν is unaffected by the scattering at BS-4.

For studying the nonlocal correlation between the two subsystems we calculate the joint detection probabilities P_{ab} of two electrons (one from lead $a \in \alpha$ or β and the other from lead $b \in \gamma$ or δ). For instance, joint-detection probability at leads α and γ is given by

$$P_{\alpha\gamma} = |r_1 r_2 u_\gamma + t_1 t'_2 v_\gamma|^2, \quad (7)$$

where the coefficient $u_\gamma \equiv r_3 r_4 + t_3 t'_4$ ($v_\gamma \equiv r_3 r_4 + t_3 t'_4 e^{i\Delta\phi}$) represents the amplitude of finding an electron at lead γ under the condition that the electron in the upper interferometer is reflected (transmitted) at BS-1.

In ballistic conductors, it is practically impossible to count electrons one by one. Instead, the single-particle and joint-detection probabilities can be obtained by measuring the average current and the zero-frequency cross correlation [4]. The output electrodes are grounded with zero voltage. The input electrodes $(\bar{\alpha}, \bar{\gamma})$ are biased by the voltage eV . We assume that the transmission of entangled electrons can be written down as an ‘entangled many-body transport state’ of the form

$$|\bar{\Psi}\rangle = \prod_{0 < E < eV} \left[r_1 b_{\alpha}^{\dagger}(E) \chi_r^{\dagger}(E) + t_1 b_{\beta}^{\dagger}(E) \chi_t^{\dagger}(E) \right] |\bar{0}\rangle, \quad (8)$$

where $|\bar{0}\rangle$ stands for the filled Fermi sea in all leads at energies $E < 0$. The main assumption made here is that the injected electrons from the two sources interact with each other and are transmitted as entangled pairs as illustrated in Fig. 1.

For the state $|\bar{\Psi}\rangle$, the output current at lead a (I_a) is proportional to the probability of finding an electron at this lead, $I_a = (e^2/h)P_a V$. The zero-frequency cross correlation, S_{ab} , of the current fluctuations ΔI_a and ΔI_b , is defined as [7]

$$S_{ab} = \int dt \langle \bar{\Psi} | \Delta I_a(t) \Delta I_b(0) + \Delta I_b(0) \Delta I_a(t) | \bar{\Psi} \rangle. \quad (9)$$

For $a \in \alpha, \beta$ and $b \in \gamma, \delta$, this cross correlator provides information about the two-particle interactions between the two interferometers which is expressed in terms of the following useful relation:

$$S_{ab} = \frac{2e^2}{h} eV (P_{ab} - P_a P_b). \quad (10)$$

Therefore, the joint-detection probabilities can be obtained by measuring the average current and cross correlation.

BI is a test to distinguish quantum correlation from local hidden variable theory by considering two-particle correlation [1]. Let us first consider the maximally entangled case: BS-1 and BS-3 are symmetric and $\Delta\phi = \pi$. General case with arbitrary $\Delta\phi$ is discussed later. BS-2 is also being kept symmetric, and the phase enclosed by the loop of the lower interferometer (ϕ_2) is fixed at $\varphi_2 = \pi/2$. The phase of the upper loop (ϕ_1) and the transmission probability of BS-4 (T_4) are controlled to test the BI. Adopting a new phase variable θ as $T_4 = \sin^2(\theta/2)$, the BI is written in the form [8]

$$S = |E(\phi_1, \theta) - E(\phi'_1, \theta) + E(\phi_1, \theta') + E(\phi'_1, \theta')| \leq 2, \quad (11)$$

where

$$E(\phi_1, \theta) = P_{\alpha\gamma} + P_{\beta\delta} - P_{\alpha\delta} - P_{\beta\gamma} = \cos(\phi_1 - \theta). \quad (12)$$

The maximal violation of the BI is found for $\phi_1 = 0, \phi'_1 = \pi/2, \theta = -\pi/4$, and $\theta' = \pi/4$, where the maximum Bell parameter is $S_{max} = \max S = 2\sqrt{2}$.

For an arbitrary value of $\Delta\phi$ (i.e., for an arbitrary degree of entanglement), we also find the maximum Bell parameter by optimizing the phases ϕ_1, ϕ_2 and θ . It reads

$$S_{max} = 2\sqrt{1 + \sin^2(\Delta\phi/2)}, \quad (13)$$

i.e., the BI is violated for nonzero $\sin(\Delta\phi/2)$. This implies that the BI is violated for any entangled pure state [9]. Note that the state of Eq.(4) cannot be written in a product state of two electrons unless $\sin(\Delta\phi/2) = 0$.

In conclusion, Bell's inequality can be tested in coupled electronic Mach-Zehnder interferometers. Entanglement of electrons is generated via Coulomb-interaction-induced modification of trajectories. This entanglement suppresses interference fringe for the single electron transport. Investigation on the two-electron correlation shows that the Bell's inequality is violated for any degree of entanglement.

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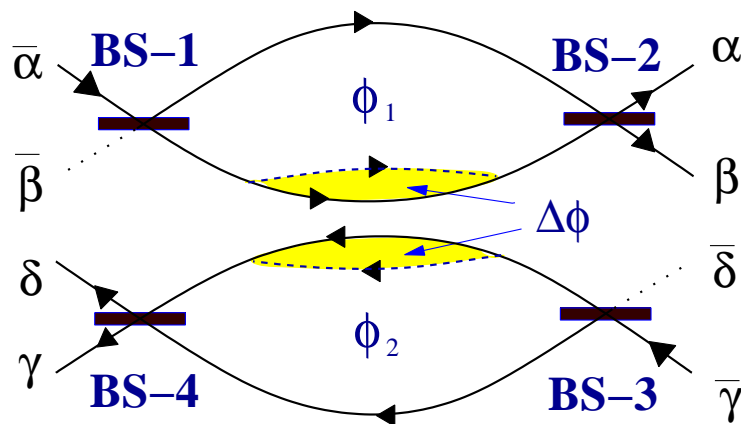


Fig. 1. Schematic figure of the coupled electronic Mach-Zehnder interferometers to test the Bell's inequality

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